

# FDTD Simulation of Multilayer-Coated and Rough Surface Metals Using Surface Impedance Method

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**Abstract** — A surface impedance method with synchronization in time and its application to the FDTD simulation of multilayer-coated metals and rough surface metals are presented. The numerical results showed that the presented method is accurate and efficient.

## I. INTRODUCTION

There are two major surface impedance methods in FDTD simulation of conductors [1]. One is the convolution based and the other is the equivalent-circuit based. The convolution-based surface impedance method is more general and can be applied to the multilayer-coated metals and rough surface metals.

In this paper, a surface impedance method with synchronization in time is presented. The method is based on the piecewise linear recursive convolution method [2] and is slightly more accurate than the original surface impedance method without synchronization.

The synchronized surface impedance method is applied to the FDTD simulation of multilayer-coated metals and rough surface metals. The surface impedance of multilayer-coated metals and rough surface metals can be found in [3]-[5]. The surface impedance can be approximated by the national models [6] before the surface impedance method is applied in the FDTD simulation. The numerical experiments showed that the FDTD simulation agrees well with the analytical solution.

## II. THE FORMULATIONS

Using the piecewise linear recursive convolution technique, the update equation for the tangential electric field on a conductor surface can be written as:

$$E_t^{n+1} = k_0 \hat{n} \times H_t^{n+1/2} + \sum_{i=1}^N \psi_i^{n+1/2}, \quad (1)$$

where  $\hat{n}$  is the surface normal,  $\psi_i^{n+1/2} = (\chi_i - \xi_i) \hat{n} \times H_t^{n+1/2} + \xi_i \hat{n} \times H_t^{n-1/2} + \rho_i \psi_i^{n-1/2}$ ,  $\chi_i = -\frac{r_i}{p_i} (1 - e^{p_i \Delta t})$ ,  $\xi_i = -\frac{r_i}{p_i^2 \Delta t} [(1 - p_i \Delta t) e^{p_i \Delta t} - 1]$ , and  $\rho_i = e^{p_i \Delta t}$ .

Here  $r_i$  and  $p_i$  are the residues and poles extracted from the surface impedance,  $\Delta t$  is the timestep and  $N$  is the number of the order. Note that the electric and magnetic fields are not collocated in space and also have half a time step offset.

To synchronize the time, the interpolation of time for the magnetic fields is applied to (1). The following update equation can be obtained:

$$H_x^{n+3/2} = \frac{1}{1 + \Delta t / \mu \Delta z} \frac{k_0 + \sum_{i=1}^N (\chi_i - \xi_i)}{2} \left\{ H_x^{n+1/2} + \frac{\Delta t}{\mu} \left( -\frac{E_z^{n+1} - E_z^{n+1}}{\Delta y} + \frac{E_y^{n+1} - E_y^{n+1}}{\Delta z} \right) \right\}, \quad (2)$$

where  $E_t^{n+1} = k_0 \hat{n} \times \frac{H_t^{n+1/2}}{2} + \sum_{i=1}^N \psi_i^{n+1}$ ,  $\psi_i^{n+1} = \chi_i \hat{n} \times \frac{H_t^{n+1/2}}{2} + \xi_i \hat{n} \times \frac{H_t^{n-1/2}}{2} + \rho_i \psi_i^n$  and  $\psi_i^n = (\chi_i - \xi_i) \hat{n} \times \frac{H_t^{n+1/2}}{2} + \psi_i^n$ .

The synchronization is applied to every timestep and is stable without any change to the stability condition. The surface impedance method can be applied to multilayer-coated metals and rough surface metals once their surface impedances are available. The surface impedance of multilayer-coated metals can be calculated analytically as follows. It is assumed that there are  $n$  layers of coatings. The layer number from the air to the lossy metal is 0 to  $n+1$ . The surface impedance can be expressed by [3]:

$$Z = \frac{\omega \mu_0}{k_1} \tanh \left\{ j k_1 d_1 + \tanh^{-1} \left[ \frac{k_1}{k_2} \tanh \left[ j k_2 d_2 + \tanh^{-1} \left[ \frac{k_2}{k_3} \tanh \left[ j k_3 d_3 + \dots \tanh^{-1} \left[ \frac{k_{n-1}}{k_n} \tanh \left( j k_n d_n + \tanh^{-1} \left[ \frac{k_n}{k_{n+1}} \right] \right) \right] \right] \right] \right] \right\}, \quad (3)$$

where  $k_i = \omega \sqrt{\varepsilon_i \mu_i}$  is the wavenumber of layer  $i$ ,  $\omega$  is the angular frequency,  $\varepsilon_i$  is the permittivity,  $\mu_i$  is the permeability and  $d_i$  is the thickness of layer  $i$ . Note that this equation is a little different from that in the original book. The coating can be various kinds of materials, normal or dispersive, dielectric or magnetic.

The rough surfaces can be represented by using the popular Hammerstad and Jensen model and causal Huray model [4], [5]. The surface impedance can be calculated using these models and the rational representations of these surface impedance can be obtained by the vector fitting technique [6].

## III. NUMERICAL RESULTS

A number of examples have been tested and a few of them are given here to demonstrate the efficacy of the approach.

### A. Smooth Metals

The reflection from a smooth metal surface is first analyzed. The conductivity is 1000 S/m. Six real poles are extracted for the frequency range of 0 – 30 GHz and are used in the presented approach. Fig. 1 shows the validation of the approach against the analytical method and surface conductivity correction [7]. It is seen that the results from surface conductivity correction at the particular frequencies agree quite well with those calculated from the analytical solution, but the results calculated from the

the presented approach agree well with the analytical solution in a broad frequency band. The approach is very accurate at low frequencies although the error increases slightly at high frequencies. This demonstrated that the surface impedance method is significantly more accurate in broadband than the regular update for a good conductor using a constant conductivity.

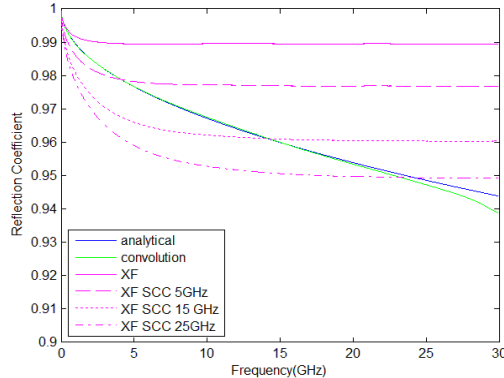


Fig. 1. Validation of convolution-based method against analytical method and surface conductivity correction (SCC) method. Note that XF means XFtd<sup>®</sup>.

### B. Coated Metals

A metal with one-layer coating is analyzed using the convolution-based methods. The surface impedance of the coated metal is fitted using six poles in the frequency range of 0 – 30 GHz. Fig. 2 shows the reflection coefficients from the coated metal. It is seen that the FDTD simulation results agree well with the analytical solution. The synchronization in time slightly improved the accuracy at high frequencies.

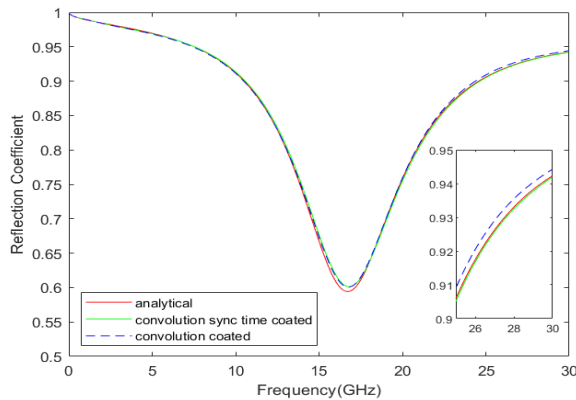


Fig. 2. The reflection coefficients from the coated metal ( $\sigma=1000$ ) with one layer of normal material (thickness  $d=0.25$  mm, conductivity  $\sigma=0.1$ , dielectric constant  $\epsilon_r=200$ ) using the convolution-based methods with and without synchronization in time and analytical method.

### C. Rough Surface Metals

Rough surface metals are also analyzed using the convolution-based methods. Fig. 3 shows the reflection coefficients from rough surface copper using the Hammerstad and Jensen model simulated by the convolution-based methods without/with the synchronization in time. The results from the FDTD simulation actually follows the analytical solution reasonably well, noting that the range of the y-axis is between 0.999 and 1. Fig. 4 shows the reflection coefficients from rough

surface copper using the causal Huray model. The results of the two rough surface models look similar.

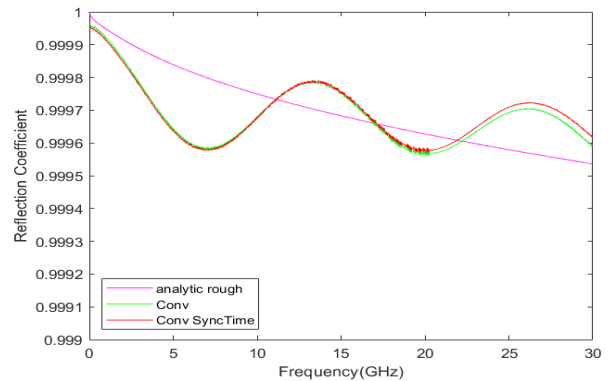


Fig. 3. Reflection coefficients from rough surface copper using Hammerstad and Jensen model (conductivity  $\sigma=5.8e7$ , roughness  $\Delta=1$   $\mu\text{m}$ ): analytical solution vs. convolution-based methods.

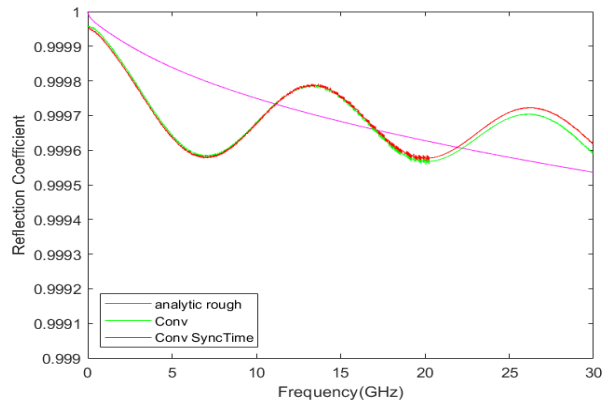


Fig. 4. The reflection coefficients from rough surface copper using causal Huray model (radius  $a=0.85$   $\mu\text{m}$ , area  $A=65$   $\mu\text{m}^2$  and number of spheres  $N=11$ ): analytical solution vs. convolution-based methods.

## IV. CONCLUSION

The surface impedance method with synchronization in time is demonstrated to be well suited for FDTD simulation of multilayer-coated metals and rough surface metals.

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